Modelling of shallow and inefficient convection in the outer layers of the Sun using realistic physics

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ABSTRACT

In an attempt to understand the properties of convective energy transport in the solar convection zone, a numerical model has been constructed for turbulent flows in a compressible, radiation-coupled, non-magnetic, gravitationally stratified medium using a realistic equation of state and realistic opacities. The time-dependent, three-dimensional hydrodynamic equations are solved with minimal simplifications.

The statistical information obtained from the present simulation provides an improved understanding of solar photospheric convection. The characteristics of solar convection in shallow regions is parameterized and compared with the results of Chan and Sofia's simulations of deep and efficient convection (Chan and Sofia 1989). We assess the importance of the zones of partial ionization in the simulation, and confirm that the radiative energy transfer is negligible throughout the region except in the uppermost scale heights of the convection zone, a region of very high super-adiabaticity.

When the effects of partial ionization are included, the dynamics of flows are altered significantly. However, we confirm the Chan and Sofia result that kinetic energy flux is non-negligible and can have a negative value in the convection zone.

Subject headings: convection—stars:interiors—Sun:atmospheric motions—turbulence

1. Introduction

To investigate the mean (horizontally and time–averaged) properties of stellar convection zones, and to test the validity of the mixing length approximation, Chan and Sofia initiated a numerical study of deep, efficient convection in a compressible and stratified layer (Chan and Wolff 1982; Chan et al. 1982; Sofia and Chan 1984; Chan and Sofia 1986; Chan and Sofia 1987; Chan and Sofia 1989). In their calculations, the effects of partial ionization and energy transfer by radiation were ignored. Among several important results in their papers were; 1) the mixing length is proportional to the pressure scale height, not to the density scale height (Chan and Sofia 1987), 2) the kinetic energy flux is non-negligible and negative over most of the layer (Chan and Sofia 1989; see also Cattaneo et al. (1991)). This implies that the mixing length approximation under-estimates the enthalpy flux, the heat transfer by convective elements, and 3) there are simple relationships between fluctuating quantities and the mean dynamical and thermodynamical structure which are similar to those given by the mixing length approximation (Vitense 1953, Böhm-Vitense 1958; Chan and Sofia 1989). Thus, certain parts of the mixing length theory were found to be valid in the limit that the convection is deep and efficient, a condition which applies to much of the solar convection zone.

Unfortunately, solar (and stellar) structure models have their greatest sensitivity to the mixing length parameter(s) in regions of partial ionization and where radiative energy transport becomes important. For example, Kim et al. (1991) show what mixing length parameters are required when changes are made to the equation of state, and alternate radiative opacities are included. Whenever opacity increases, we need to increase the efficiency of convective energy transport by increasing the mixing length ratio. Note, however, that different mixing length ratios did not cause much structural change in most of convection zone where the $\nabla - \nabla_{ad}$ is negligible.

In the shallow layers of stars, regions of partial ionization provide an additional driving (and perhaps damping) force for convection motions. To date, few quantitative modeling efforts have addressed this issue (see Rast and Toomre 1993ab, Rast et al. 1993). Furthermore, information on convection in the convective-radiative transition layers is vital to many branches of solar physics (solar dynamo, solar variability, solar atmosphere modeling, etc.) in addition to the general study of stellar structure. It is not *a priori* clear to what degree the models of deep convection can represent such layers. In this paper, we investigate whether the results from studies of deep and efficient convection still hold in a shallow and inefficient convective zone, where the convection becomes less efficient and where ionization plays important role in the dynamics of convection.

The numerical model for solar photospheric convection in a compressible, radiation-coupled, non-magnetic, gravitationally stratified medium is described in section 2.

By performing statistical analyses of the simulation, we determine properties of solar convection, and compare them with those obtained by Chan and Sofia(1989), in section 3. In section 4, the results of this study are summarized and discussed.

2. Convection simulation

2.1. Hydrodynamical equations

Consider a fluid moving in a known gravitational field g, carrying no electric charge or electric current, and undergoing no chemical reactions. In a non-rotating frame of reference, the equations representing this stratified compressible hydrodynamical flow in three dimensions are:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{M}, \tag{1}$$

$$\frac{\partial \mathbf{M}}{\partial t} = -\nabla \cdot \left[\mathbf{M}\frac{\mathbf{M}}{\rho} - \boldsymbol{\Sigma} + P\mathbf{I} \right] + \rho\mathbf{g}, \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot \left[(\varepsilon + P)\frac{\mathbf{M}}{\rho} - \frac{\mathbf{M}}{\rho} \cdot \boldsymbol{\Sigma} + \mathbf{f} \right] + \mathbf{M} \cdot \mathbf{g}, \tag{3}$$

$$\varepsilon = e + \frac{1}{2}\rho\mathbf{v}^2, \qquad \text{Total Energy} \tag{4}$$

$$P = P(e, \rho, \mu), \qquad \text{Pressure} \tag{5}$$

$$T = T(e, \rho, \mu), \qquad \text{Temperature} \tag{6}$$

$$\boldsymbol{\Sigma} = 2\bar{\mu}\sigma + \lambda(\nabla \cdot \mathbf{v})\mathbf{I}, \qquad \text{Viscous Stress Tensor} \tag{7}$$

where ρ is the density, $\mathbf{M}$ is the mass flux vector ($\rho\mathbf{v}$), e is the specific internal energy, μ is the mean molecular weight, $\mathbf{g}$ is the gravitational acceleration, $\mathbf{I}$ is the identity tensor, and σ is the usual rate of strain tensor ($\sigma_{ij} = \frac{(\nabla_i v_j + \nabla_j v_i)}{2}$).

The goal of the direct simulations of stellar convection is to solve the equations governing compressible fluid dynamics in a star with the fewest possible approximations. Some approximations, however, are unavoidable. One such assumption concerns the resolution of the simulation which models flows with large Reynolds numbers and large Rayleigh numbers.

It is not possible with currently available computer power to encompass all scales which are present in stellar convection. In most stellar convection zones, the effective Rayleigh numbers (Ra) and the Reynolds numbers (Re) are very large. Since the ratio between a pressure scale height and the Kolmogorov microscale, below which the flow really becomes smooth, is of order $Re^{\frac{3}{4}} \approx 10^{7.5}$, $Re^{\frac{9}{4}} \approx 10^{22.5}$ grid points are required to resolve the motions in all scales. Even if advances in computer capabilities made such a simulation feasible, we may not be interested in such fine resolution, since the smallest scales do not participate in the heat transport.

In any case, current hydrodynamic simulations calculate only the larger scales of the

convective flow, and use some prescription to take into account the smaller, subgrid scales. The simplest way is to choose a fixed, larger value of the viscosity to represent the effect of the Reynolds stresses due to the subgrid scales, assuming that the scale is within an inertial cascade range in the local turbulence spectrum. Since our grid spacings and the actual Reynolds number were not uniform, however, we employed a variable, also larger, viscosity which raises the local effective grid Reynolds number to order unity everywhere (Ramshaw 1979). Instead of using the gas viscosity values, the forms of $\bar{\mu}$ and λ in equation (7) are chosen to represent the effects of the Reynolds stresses on unresolved scales (Smagorinsky 1963):

$$\bar{\mu} \;=\; \rho \left(c_\mu \Delta \right)^2 \left(2\sigma : \sigma \right)^{\frac{1}{2}}, \qquad \text{Dynamic Viscosity} \tag{8}$$

where the Deardorff coefficient $c_\mu = 0.2$ was used. Ignoring bulk viscosity, the quantity λ was taken to be $-2/3\bar{\mu}$ (Stoke's hypothesis).

The total heat flux $\mathbf{F}$ is made up of $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_k + \mathbf{F}_v + \mathbf{f}$ where the enthalpy, kinetic energy, viscous and diffusive fluxes respectively are:

$$\mathbf{F}_e \;=\; (e + P)\frac{\mathbf{M}}{\rho} \tag{9}$$

$$\mathbf{F}_k \;=\; \left(\frac{1}{2}\rho V^2 \right) \mathbf{V} \tag{10}$$

$$\mathbf{F}_v \;=\; -\mathbf{V} \cdot \mathbf{\Sigma} \tag{11}$$

$$\mathbf{f} \;=\; -K_T \nabla T + K_p \nabla P, \tag{12}$$

and thermal conductivities are:

$$K_T \;=\; \frac{\mu}{Pr} C_p + \frac{4acT^3}{3k\rho} \tag{13}$$

$$K_p \;=\; \frac{\mu}{Pr} C_p \nabla_{ad} \frac{T}{P}, \tag{14}$$

where c_p is the specific heat at constant pressure, ∇_a is the adiabatic gradient $\equiv \frac{\partial \ln T}{\partial \ln P}\big|_{ad}$, and $Pr = \nu/\kappa$ is the Prandtl number (ν is the kinematic viscosity and κ is the thermal

diffusivity). Radiative transfer is treated with the diffusion approximation, where k is the opacity, a is the Boltzmann constant and c is the speed of light. When the mean free path of a photon is much shorter than the depth of the zone, the radiative diffusion treatment is a good approximation for the equation of radiative transfer. Therefore, in the most of our calculation domain, the approximation is expected to be valid. At the top, however, the photon mean free path is $\frac{1}{10}$ of the depth. It is not clear that the approximation is still valid in the hoped-for degree. — The photon mean free path ($\frac{1}{\kappa\rho}$) is about $9 \times 10^6 cm$. The top of the calculation domain locates at the depth of $9 \times 10^7 cm$. The top of the solar model is defined where the density is $10^{-10} g cm^{-3}$.—

2.2. The numerical solution; ADISM

The equations governing the fluid motion form a system of coupled non-linear partial differential equations. Although this system is based on the very simple physical principles of conservation of mass (1), momentum (2), and energy (3), analytical solutions to this system of equations can only be found under some very specific and often unrealistic assumptions (special symmetry, small perturbations, etc.). Therefore, for physically realistic solutions, this system of equations must be solved numerically.

To study large scale convection in the Sun, the system of the hydrodynamic equations was solved using the 'Alternating Direction Implicit on Staggered Mesh' method (ADISM; Chan and Wolff 1982). This method was accurate to second order even for non-uniform zoning (staggered mesh), free from the time step restrictions for stability (implicit method), and efficient for solving multi-dimensional hydrodynamics equations (Alternating Direction method). The code used for this research was a three-dimensional and non-magnetic version of the code Fox developed (Fox et al. 1991), using Chan's GENTRX code (Chan

and Wolff 1982).

While the grid spacing was uniform in the horizontal direction, the spacing in the vertical direction was chosen to be non-uniform, so that higher resolution can be obtained near the top of the calculation domain where scale heights are small. The ith layer is located at

$$
r_i \;\; = \;\; \frac{r_2}{1 + \frac{(r_2 - r_1)}{Z_G r_1} \{ \exp[\frac{N_r - i}{N_r - 1} \ln(1 + Z_G)] - 1 \}},
\tag{15}
$$

where r_1, r_2 are the bottom and top vertical coordinates, respectively, N_r is the number of grids in the vertical direction, and Z_G is the controlling parameter of the vertical grid distribution (Chan and Sofia 1986).

To control numerical truncation errors caused by the nonuniform vertical gridding, a coordinate transformation has been introduced in such a way that the mapped grids are uniform, and all the quantities are made dimensionless so that the initial values of the density, temperature, and pressure at the top are equal to 1.

2.3. Initial conditions

A solar model is constructed using the Yale stellar evolution code (Guenther et al. 1989, Kim et al. 1991, Guenther et al. 1992) with solar abundances given by Anders-Grevesse (1989) and opacities from the Los Alamos Opacity Library (LAOL) tables (Huebner et al. 1977).

The equation of state is divided into two separate regions and an intermediate transition region. In the outer region, the equation of state routine determines particle densities by solving the Saha equation for the single ionization state of H and heavier elements, and the single and double ionization states of He. Perturbations due to Coulomb interactions

and excited states in bound systems are neglected. In the inner regions, the elements are assumed to be fully ionized. The partially degenerate, partially relativistic case is handled with an iteration method. In the transition region, the interior and exterior formulation are weighted with a ramp function and averaged.

The solar He mass fraction, Y (which cannot be determined by observation), together with the mixing length parameter, α, are free parameters in stellar model construction. As is usual with the given model physics, α and Y were adjusted in such a way that a solar model has the solar radius and the solar luminosity at the solar age (Guenther 1989). The characteristics of the model are summarized in table (1).

The stratifications of the internal energy, the density, and the gravity of the photospheric convection zone in the model were then taken as the initial condition for the hydrodynamic simulation. This ensures consistency between the shallow regions where the photospheric convection was simulated and the deeper regions of the stellar structure. The initial condition used for the simulation is shown in figure 1. Since the hydrodynamic equations are solved in nondimensional form, the scaling factors are obtained from the surface level of the solar models and are listed in table (2).

A small kinematic perturbation was applied to initiate motion:

$$M_x = V_0 \rho \sin \theta_x \left(-\cos \theta_z \sin \theta_y + \sin \theta_z \cos \theta_y \right), \qquad (16)$$

$$M_z = -V_0 \rho \cos \theta_x \sin \theta_z \left(-\cos \theta_y + \sin \theta_y \right), \qquad (17)$$

$$M_y = V_0 \rho \sin \theta_y \left(\cos \theta_x \sin \theta_z - \sin \theta_x \cos \theta_z \right), \qquad (18)$$

where

$$\theta_i = \frac{N_i(grid\ position)\pi}{number\ of\ grid\ spacings}.$$

Table 1: Solar model

i represents the spatial coordinates, V_0 is the amplitude of the initial perturbation, and N_i controls the number of cells in the initial perturbation.

2.4. Boundary conditions

The computational domain was a rectangular box with impenetrable, stress-free boundaries at the top and bottom. A constant heat flux, which was calculated from the hydrostatic model structure, was imposed at the bottom. The input flux is shown as $\frac{F}{F_T}$ in table (2). Thus:

$$\mathbf{V} \cdot \mathbf{n} = 0, \qquad \frac{\partial}{\partial n} \left(\mathbf{V} \times \mathbf{n} \right) = 0, \tag{19}$$

$$F_z = F_{bottom} = \text{constant (at bottom only)}, \tag{20}$$

$$e = e_{top} = \text{constant (at top only)}. \tag{21}$$

Periodic boundary conditions for all variables were used in the horizontal direction.

2.5. Simulation specifications

The hydrodynamic equations (conservation of mass (1), momentum (2), and energy (3)), together with the same equation of state and opacities which were used in the initial model constructions, were solved in time, until a converged statistically steady-state solution was reached. To control the time step of the simulations, a dimensionless number was defined:

$$N_{CFL} = \frac{\Delta t C_s}{\Delta r},$$

Table 2: Scaling factors associated with the initial condition.

where C_s is the sound speed at the top, Δr is the minimum grid spacing, and Δt is the time step. Since our mesh system is nonuniform in the sense that we can obtain higher resolution at the top, Δr in this case is usually Δz at the top.

Further conditions for the simulations are summarized in Tables (3) and (4).

To determine whether a calculation has 'relaxed', the time history of the horizontally averaged surface heat flux and the maximum RMS velocity in the domain were monitored.

In figure 2, 3, and 4, density and temperature contours and velocity fields of the simulation are shown at a time after the statistically steady-state was reached. To visualize the three-dimensional flow pattern, contours on one x-z plane, one y-z plane, and three different levels of x-y plane are shown.

The left-top panel in figure 4 shows the flow pattern which resembles the solar granules and intergranule regions. The topology change with depth is shown at the left three panels in figure 4. The relative density fluctuations and the relative temperature fluctuations are higher at the top.

The tight correlation between the radial velocity and the temperature and the density fluctuations at the top found in other studies (Chan and Sofia 1986, 1989) is also evident in these simulations. For example, the anti-correlation between the radial velocity and the density fluctuation is -0.8 or greater.

While these figures are useful in obtaining the gross character of the simulated convection, more physical information can only be extracted from the simulations using statistical quantities. For example, in the case of stars, the observed spectral absorption line asymmetries only provides temporally and spatially integrated information on the surface

Table 3: The simulation specification.

convective motions. Even for solar convection, which is the only case where time-resolved information can be obtained, the overall characteristics, not detailed temporal variations, are of interest in this paper. From the detailed analysis of the statistical information, the validity of prescriptions of convective energy transport, such as the mixing length approximation, can be tested.

3. Statistical properties

Statistical averages of the simulation are calculated to study the general properties of solar convection. The simulation carried out corresponds to 2.08×10^5 seconds of solar convection. The horizontal and temporal averages of the quantities of interest were calculated during a period of about 16 turnover times (7.11×10^4 seconds in the simulation). The characteristics of convection were parameterized by relationships suggested both by mixing length theory and the results of Chan and Sofia (1989), and are summarized in table 5. These results were calculated using the middle 4 pressure scale heights of the layer, where the mean vertical mass flux $\langle \rho V_z \rangle \leq 1 \times 10^{-4}$, implying that the mean distribution of the fluid was not undergoing substantial re-adjustment.

For a quantity a, $\langle a \rangle$ denotes the combined horizontal and temporal mean, a' denotes the deviation from the mean, and a'' denotes the root mean square fluctuation from the mean. The correlation coefficient of two quantities a_1 and a_2 is expressed as $C[a_1, a_2]$:

$$C[a_1, a_2] = \frac{\langle a_1 a_2 \rangle}{\sqrt{\langle a_1{}^2 \rangle}\sqrt{\langle a_2{}^2 \rangle}}.$$

All quantities in the tables are scaled values. (See table 2).

Table 4: The initial perturbation.

In Table 5, our results are compared with those of Chan and Sofia (1989). At this stage, it is necessary to restate some difference between the physical conditions in each study. Since their interest was in deep, efficient convection (Chan and Wolff 1982; Chan et al. 1982; Sofia and Chan 1984; Chan and Sofia 1986; Chan and Sofia 1987; Chan and Sofia 1989), the effects of ionization and energy transfer by radiation were ignored. Since our interest is in the shallower layers, these effects are present in our simulation and thus are expected to produce different parameterizations. For the present simulation, it turned out that the effect of radiative energy transport was quite small throughout the calculation domain, except in the uppermost scale height. Therefore, in most regions in the solar convection zone, ignoring the coupling between radiation and convection seems to be a reasonable approximation. (More discussion regarding the effect of radiative energy transfer can be found later together with the distribution of fluxes.)

Chan and Sofia (1989) used a unit constant mean molecular weight and, by changing the ratio of specific heats, they calculated convection of gases with different ionization stages. Since our simulations used an equation of state in which the ionization of H and He was calculated based on the local thermodynamic conditions, the mean molecular weight varied temporally and spatially. The temporal and horizontal average of the mean molecular weight is shown in figure 5.

Finally, because our initial structure was obtained from a solar model, the simulation domain contained a highly super-adiabatic region as shown in figure 6, near 12.5 in logarithmic pressure. The Chan and Sofia simulations used an initial polytropic structure model and thus this feature was absent.

From Table (5), a few characteristic differences can be found. The horizontal velocity

Table 5: Approximate relationships obtained from the numerical simulations.

fluctuation is comparable to the vertical velocity fluctuation, except near the top and bottom where the boundary condition enforces V_z'' to be zero and the horizontal velocities are large. This seemingly isotropic velocity fluctuation, shown in figure 7, did not exist in the calculations of Chan and Sofia (1989).

The relative fluctuations of the thermodynamic variables $\rho''/\langle \rho \rangle$, $T''/\langle T \rangle$, and $P''/\langle P \rangle$, shown in figure 8, seem to indicate that the ratio between the three fluctuation quantities changes in the highly super-adiabatic region. It is, however, difficult to judge whether the different characteristics are from the degree of super-adiabaticity or the top boundary condition. As shown in figure 6, the relaxed solution has the pressure enhanced at the top. Even in the regions where $(\nabla - \nabla_{ad})$ was close to zero, the ratios between the fluctuating quantities are different from that of Chan and Sofia (1989). In addition, $P''/\langle P \rangle$ and $\rho''/\langle \rho \rangle$ are larger than $T''/\langle T \rangle$. Therefore, the contribution of the pressure term in the entropy fluctuation $\delta S = C_p(\delta \ln T - \nabla_{ad} \, \delta \ln P)$ is not negligible, in contrast to their result.

In the present simulations, it was found that the relative pressure fluctuation was proportional to the square of the Mach number. The coefficient of the relation between $P''/\langle P \rangle$ and $\mu V''^2/\langle T \rangle$, however, was different (Table 5).

The relation between P'' and $\rho V_z''^2$ was derived from the relation between $P''/\langle P \rangle$ and $\mu V''^2/\langle T \rangle$ and the relative ratio between the fluctuations of the thermodynamic parameters. Since the relative ratios among the fluctuating quantities are different in the highly super-adiabatic region, as shown in figure 8, the correlation is not as tight as in the calculation of Chan and Sofia (1989).

The mean values of the correlation coefficients between pairs of parameters are shown in Table 5 and in figure 9 and 10. While others correlations in Table 5 are close to what was found for deep efficient convection, $C[\rho', T']$, $C[P', T']$, and $C[P', T']$ are noticeably different.

Further relations for the covariance of V_z with the thermodynamical variables are calculated to compare with those of Chan and Sofia (1989). The covariance $\langle V_z \rho' \rangle / \langle V_z \rangle \langle \rho \rangle = -1$ implies simply mass conservation, $\langle \rho V_z \rangle = 0$. Therefore, this indicates that the mean distribution of the fluid in the simulation is no longer undergoing substantial adjustment. The covariance, $\langle V_z P \rangle / \langle V_z \rangle \langle P \rangle \approx 1.75$, which implies $\langle V_z P' \rangle / \langle V_z \rangle \langle P \rangle \approx 0.75$, and the covariance $\langle V_z T' \rangle / \langle V_z \rangle \langle T \rangle \approx 0.85$, implies that P'/P, and T'/T are not negligible in the convective flow.

The characteristic differences in the properties of convection in this research and in Chan and Sofia (1989) can be summarized as follows: unlike their deep efficient convection, the velocity fluctuations were close to isotropic and the pressure and density fluctuations are larger than the temperature fluctuations; the correlations among T', P', and ρ' found in this research are quite different from those in Chan and Sofia (1989). It appears that the inclusion of ionization was responsible for these differences. In a perfect gas, the temperature change causes the density change. When there is a strong correlation between these two variables, the correlation of these parameters with pressure must be low. This is what we observe from Chan and Sofia's simulation. On the other hand, when the temperature change is not directly connected to the density change, because of ionization, the correlation of these parameters with pressure must be stronger. Since ionization changes the number of light particles (electrons), the pressure change will be larger than the density change. This simple explanation helps one to understand the differences in the characteristics of convection in this research and in Chan and Sofia (1989).

The differences between convection in the two studies can also be shown in another way. From their deep efficient convection simulations, Chan and Sofia (1987, 1989) showed that the vertical correlation length of vertical velocity and temperature deviation are scaled by the pressure scale height, not by the density scale height. In their research, they calculated

several cases with different ratios of specific heats, γ. The practical meaning of different γ's is different mean molecular weights. Therefore, each calculation with a different γ can 'simulate' the convection of gases at different ionization stages. When the vertical two-point correlations of V_z from several simulations with different γ's were plotted together, they scaled with pressure scale height, not with density scale height. When the partial ionization process is coupled with the convection calculation, the situation is changed. In figure 11 and 12 the vertical, two-point correlations at five different depths are plotted together. As shown in figure 5, since the average mean molecular weight varies as a function of depth, it practically means each symbol corresponds to a case with different γ. The vertical two-point correlation of V_z can be scaled with the density scale height as well as with the pressure scale height (cf. Chan and Sofia 1987). The vertical correlation of T', however, can not be scaled with pressure scale height, nor density scale height.

Yet another difference is apparent in the distribution of energy fluxes. The distribution, shown in figure 13 basically confirms the study of Chan and Sofia (1989). The kinetic energy flux was not negligible in the convection zone, and it was negative. This important result holds in shallow region convection as well as in deep efficient convection. The enthalpy flux is larger than the total flux. Recall that in the mixing length approximation, the total flux is carried by two components only, radiation and convection, the latter is equivalent to the enthalpy flux and thus is under-estimated (see the discussion in Lydon et al. 1992). This result casts further doubts on the mixing length approximation in the shallow convective regions of a star.

Compared with Chan and Sofia (1989), however, the diffusive flux is much larger. When zones of partial ionization overlap with the convection zone, a certain amount of the energy flux is taken away for ionization so that the same amount of energy need not be transferred by means of convection. Practically speaking, C_p changes in the diffusion term

in the energy equation, equation (3). An increase in C_p means an increase in the thermal conductivity (see equation (12), (13), and (14)). This larger diffusive action may explain the seemingly isotropic velocity fluctuation discussed earlier. Chan and Sofia (1989) found the vertical velocity fluctuations are larger than the horizontal velocity fluctuations in their convective flows.

The effect of the radiative energy transport in the convection zone is apparent in figure 13. The radiative energy flux (the dot-dash line) is small throughout the region except at the top one pressure scale height. This result confirms the assumption used for the simulations of deep efficient convection by Chan and Sofia (1989): The radiative flux can be assumed to be negligible for deep efficient convection.

One must remember that this simulation has a rigid top and bottom boundary condition. Therefore, flows which impinge upon the boundary have to diverge. As a result, the V_z is smaller close to the top and bottom boundary. This causes the enthalpy flux and the kinetic energy flux to be small at the bottom. Only the diffusive flux can be increased to carry the flux out at the bottom. At the top, the radiative flux takes over the transfer of the energy flux.

There are two additional comments for figure 13. Firstly, at the top the radiative flux exceeds the total flux, and the sub-grid scale diffusive flux becomes negative (i.e. an entropy inversion). This indicate that the 'mean' structure in the region has become stable to convection. This conclusion must be viewed cautiously since this effect happens at the very top of the vertical grid and may be an un-physical artifact of the top boundary condition.

Secondly, at the bottom the kinetic energy flux is positive, whence through out most of the region it is negative. Because of the rigid bottom boundary, the characteristic flow pattern is modified so that down-ward flows can not but diverge and larger horizontal velocities are induced. As a result, the kinetic energy flux can not be negative.

In summary, our research confirms one assumption which Chan and Sofia employed for their study: that radiation can be de-coupled from convection in regions where the motions are deep and efficient. The contribution of the radiative flux in our simulation became significant only at the top one pressure scale height. The partial ionization process, however, should not be de-coupled from convection in the shallow layers. As shown in figure 7 through 13 and summarized in table 5, the effects of partial ionization is not negligible.

4.   Summary

Three-dimensional hydrodynamic simulations for the shallow layers of the solar convection zone have been performed in order to probe the general properties of convective energy transport. Our simulations were made using a realistic equation of state and opacities and allowed for radiative energy transport in the diffusion approximation. To ensure the overall consistency of the structure, the initial model for the convection simulation was taken from a detailed solar structure model constructed using the Yale stellar evolution code.

From the detailed analysis of the simulation, the characteristics of solar convection in shallow regions have been parameterized and compared with that of Chan and Sofia's deep, efficient convection simulations (Chan and Sofia 1989). From the differences between the statistical properties obtained, two important points were drawn. First, the transition from the radiative mode to the convective mode of energy transport occurs at a very thin layer at the top of the convection zone. The contribution of energy transport by radiation in the simulation was negligible throughout most of the domain except within the top one pressure scale height. Therefore, it is safe to de-couple the radiation from convection simulation except in super-adiabatic regions.

Secondly, the thermodynamic effects of partial ionization on the dynamics of the flow were significant and led to different statistical properties compared to the study of Chan and Sofia which used a fully ionized gas with different specific heats. Therefore if the emphasis of a simulation is on detailed comparison with solar surface features (for example), every effort should be made to include a realistic equation of state.

Comparison with three of the results of Chan and Sofia deserve to be pointed out especially. Firstly, unlike the result of Chan and Sofia (1987, 1989), our study shows that the pressure scale height is not a preferable scaling factor over the density scale height in the shallow convective layers. Secondly, in agreement with Chan and Sofia(1989), we find that the kinetic energy flux in the upper convective zone is not negligible, and that it is mostly downward directed. As a result, the temperature gradient which the mixing length approximation prescribes, will not be appropriate for that part of the convection zone. Finally, they have shown that the mixing length approximation is valid in the limit of deep and efficient convection; we will test this result in a forth-coming paper.

Fig. 1.— The hydrostatic structure constructed using Yale stellar evolution code was used as the initial condition for the hydrodynamic simulation of the Sun.

Fig. 2.— The density contours. This snapshot of the solar convection simulation is at time step 94000. At the left column from top to bottom, density contours are on x-y plane at z=27, 16, and 5, respectively. The contour on x-z plane at y=16 grid is shown at right-top panel. The right-bottom panel shows the contour on y-z plane at x=16.

Fig. 3.— The temperature contours at the same time step as the figure 2. At the left column from top to bottom, the temperature contours are on x-y plane at z=27, 16, and 5, respectively. The contour on x-z plane at y=16 grid is shown at right-top panel. The right-bottom panel shows the contour on y-z plane at x=16.

Fig. 4.— The velocity fields at the same time step as the figure 2. At the left column from top to bottom, the velocity fields are on x-y plane at z=27, 16, and 5, respectively. The projection of the velocity vectors on the x-y plane and the contours of V_z are shown. The solid-line and the broken-line contours represent positive and negative radial velocity, respectively.
The fields on x-z plane at y=16 grid is shown at right-top panel. The projection of the velocity vectors on the x-z plane and the contours of V_y are shown. The solid-line and the broken-line contours represent positive and negative V_y, respectively.
The right-bottom panel shows the fields on y-z plane at x=16. The projection of the velocity vectors on the y-z plane and the contours of V_x are shown. The solid-line and the broken-line contours represent positive and negative V_x, respectively.

Fig. 5.— The temporal and horizontal average of the mean molecular weight calculated from the simulation.

Fig. 6.— The temporal and horizontal average of $(\nabla - \nabla_{ad})$ calculated from the simulation is represented by the crosses. The solid line represents the initial stratification. The stratification of the simulation is steeper than that of the initial model.

Fig. 7.— Root mean square velocity fluctuations versus depth. In this figure, like all others, the depth is specified by $\ln(P/P_{top})$, where P_{top} is the pressure at the top of the thermally relaxed fluid. The solid, the dashed, and the dotted lines show V_z'', V_x'', and V_y'' respectively.

Fig. 8.— Distributions of the relative fluctuations of the thermodynamic variables. The solid, the dashed, and the dotted lines are the temperature, pressure, and density fluctuations, respectively.

Fig. 9.— Correlation between fluctuating quantities. The solid, the dashed, the dash-dot, and the dotted lines are $C[T', S']$, $C[P', T']$, $C[V_z, T']$, and $C[V_z, S']$, respectively.

Fig. 10.— Correlation between fluctuating quantities. The solid, the dashed, and the dotted lines are $C[\rho', S']$, $C[V_z, \rho']$, and $C[\rho', T']$, respectively.

Fig. 11.— Distributions of the two-point correlation of vertical velocities are plotted against the logarithmic pressure (HWHM $\sim$ 1.5) and the logarithmic density (HWHM $\sim$ 1.2) for different depths.

Fig. 12.— Distributions of two-point correlation of temperature fluctuation are plotted against the logarithmic pressure and the logarithmic density for different depths.

Fig. 13.— Distribution of heat fluxes. The total flux (the thick solid line) is the summation of the enthalpy flux (the thin solid line), the kinetic energy flux (the dashed line), the viscous flux (the dotted line), the radiative flux (the dot-dash line), and the diffusive flux (the dot-dot-dot-dash line).

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